2. $F(s) = \dfrac{s^3+2s^2+s-1}{(s-1)^2[(s+1)^2+4]}$

$$\frac{s^3+2s^2+s-1}{(s-1)^2[(s+1)^2+4]} = \frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{C(s+1)}{(s+1)^2+4}$$ (b)

after evaluating $(C(s+1))$ the independent C is treated as constant D

5. $f(t) = \mathcal{L}^{-1}\left\{\dfrac{6}{(s-2)^3} + \dfrac{s-1}{(s-1)^2+4}\right\}$

$= \mathcal{L}^{-1}\dfrac{6}{(s-2)^3} + \mathcal{L}^{-1}\left[\dfrac{s-1}{(s-1)^2+4}\right]$

$= 3\mathcal{L}^{-1}\dfrac{2}{(s-2)^3}$ and this conforms to the laplace transform of $t^n e^{at}$ which is $\dfrac{n!}{(s-a)^{n+1}}$

$= 3t^2e^{2t} + \mathcal{L}^{-1}\dfrac{s-1}{(s-1)^2+4}$ The second part conforms to laplace transform of $e^{at}\cos bt$ which is $\dfrac{(s-a)}{(s-a)^2+b^2}$

$f(t) = 3t^2e^{2t} + e^{t}\cos 2t$ (a)

Scanned with CamScanner

9. $f(t) = \mathcal{L}^{-1} \dfrac{-s+6}{(s-2)(s+2)}$

By partial fraction's

$$\frac{-s+6}{\cancel{(s+2)}(s-2)} \cancel{(s+2)} = \frac{A}{\cancel{(s+2)}} \cancel{(s+2)} + \frac{B}{s-2} (s+2) \Big|_{s=-2}$$

Using cover up rule

evaluating A at $s=-2$.

$$\frac{-s+6}{s-2} = A$$

$$A = \frac{+2+6}{-2-2} = \frac{8}{-4} = -2$$

evaluating B at $s=2$.

$$\frac{-s+6}{(s+2)\cancel{(s-2)}} \cancel{(s-2)} = \frac{A}{(s+2)}(s-2) + \frac{B}{\cancel{(s-2)}} \cancel{(s-2)} \Big|_{s=2}$$

$$\frac{-s+6}{s+2} = B$$

$$B = \frac{-2+6}{2+2} = \frac{4}{4} = 1$$

$$\frac{-s+6}{(s+2)(s-2)} = \frac{-2}{(s+2)} + \frac{1}{s-2}$$

$$f(t) = \mathcal{L}^{-1} \frac{-2}{s+2} + \mathcal{L}^{-1} \frac{1}{s-2}$$

$$= -2e^{-2t} + e^{2t}$$

$$= e^{2t} - 2e^{-2t} \quad \text{(b)}$$

Scanned with CamScanner

11. $\mathcal{L} \cosh(\alpha t) = \dfrac{s}{s^2-\alpha^2}$

$f(t) = e^{2t} \cosh(4t) \qquad \alpha = 4$

$\mathcal{L} \cosh(4t) = \dfrac{s}{s^2-4^2}$

but $f(t) = e^{2t} \cosh(4t)$

$\mathcal{L}\ f(t)$ where $f(t) = a \cdot b = \mathcal{L}a \cdot \mathcal{L}b$

$= \dfrac{1}{s-2} \cdot \dfrac{s}{s^2-4^2}$ multiply denom by $(s-2)$

$= \dfrac{(s-2)\cdot s}{(s-2)^2\,(s^2-4^2)}$

$= \dfrac{s\,(s-2)}{(s^2-4s+4)(s^2-4^2)}$

$= \dfrac{s-2}{(s-2)^2+4^2}$

7. $y'' + 5y' + 6y = (2t^2+t+1)e^t \cos t$

Sol

Particular sol- has the form $y = \sum_{j=0}^{n} a_j n^j\ )e^{k \cdot n} \cos(mn) + \sum_{j=0}^{n} b_j n^j e^{kn} \sin(mn)$

So here $y'' + 5y' + 6y = (2t^2+t+1)e^t \cos t$.

$n=2, \quad m=1 \quad k=1$

$y(p)\,t) = (A_2t^2 + A_1t + A_0)e^t \cos(t) + (B_2t^2+B_1t+B_0)e^t \sin(t)$

Answer : None of the above.

12. $\frac{s^2-s+3}{(s+1)(s^2-2s+2)}$

$$\frac{s^2-s+3}{(s+1)(s^2-2s+2)} = \frac{A}{(s+1)} + \frac{Bs+C}{s^2-2s+2}$$

$$s^2-s+3 = (s+1)(Bs+C) + (s^2-2s+2)A$$

$$s^2-s+3 = s^2A + s^2B - 2sA + sB + sC + 2A + C$$

$$s^2-s+3 = s^2(A+B) + s(-2A+B+C) + 2A + C$$

$A+B=1$

$-2A+B+C=-1$

$2A+C=3$

$A=1 \quad B=0 \quad C=1.$

Therefore $\frac{s^2-s+3}{(s+1)(s^2-2s+2)} = \frac{1}{s+1} + \frac{1}{s^2-2s+2}$

solving s^2-2s+2 to make it a perfect square

$$s^2-\left(\tfrac{2}{2}\right)^2 s+\tfrac{2}{2}+2 = (s-1)^2+1.$$

Therefore $\frac{s^2-s+3}{(s+1)(s^2-2s+2)} = \frac{1}{s+1} + \frac{1}{(s-1)^2+1}$

6. $\cos 2x = \cos^2 x - \sin^2 x$

$F(s) = \mathcal{L}^{-} \cos^2(2t) - \sin^2(2t)$

$\cos^2(2t) - \sin^2 2t = \cos 4t \qquad a=4$

$\mathcal{L}\cos 4(t) = \frac{4}{s^2+16}$ since it is equal to $\frac{s}{s^2+a^2}$